# Mathematical Reasoning

Too little attention is being given to mathematical reasoning.

Too many students are unable to solve Nonroutine problems.

Students become procedurally oriented.

# Mathematical Reasoning

**What number does 11 tens, 8 ones, and 2 hundreds make?**

**Responses: 1182; 2118; 118. 02; 318**

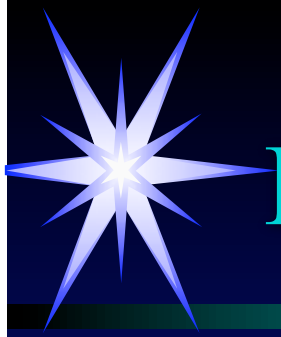
| | Level 3 |
|---|---|
| **Grade six** | **43%** |
| **Grade seven** | **46%** |
| **Grade eight** | **50%** |

**2,000+ were not successful.**

# Mathematical Reasoning

**Jill had 23 candies. She put the same number in each of two bags and had seven candies left over. How many did she put in each bag?**

| | Level 3 |
|---|---|
| Grade six | 68% |
| Grade seven | 69% |
| Grade eight | 70% |



# Problem Solving

DEFINITION:

Problem solving is what you do when you don't know what to do.

If you know how to get an answer, it is not problem solving

# Problem Solving Strategies

- A variety of strategies are needed
- Students must develop their own strategies
- When stuck, it is important to

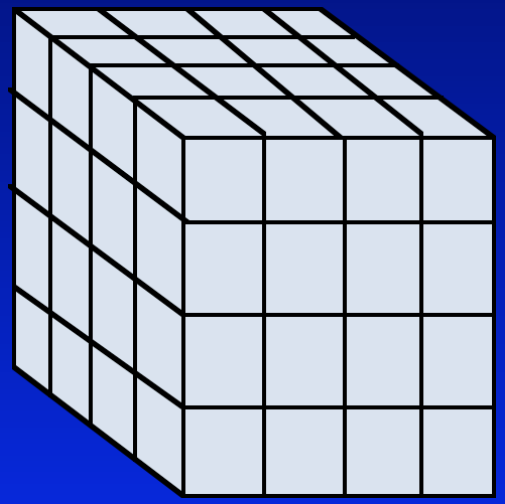
  DO SOMETHING

# Problem Solving

Problem solving now appears in all standards

- Strand
- Way of Teaching

**Which cubes share exactly four faces with other cubes?**



**How many are there?**

# Problem Solving Strategies

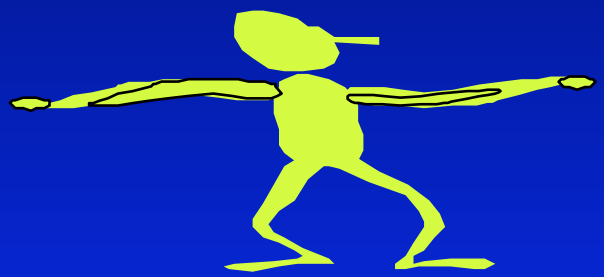
- Try a number
- Look for patterns
- Guess and test
- Draw a diagram
- Work backwards
- Act it out
- . . . . .

# Problem Solving

Fourth Grade: Find all the factors of 300.

"I know that between a hundred and a hundred fifty, none of those can work because a hundred is, goes into three hundred three times, and a hundred and fifty goes in two times, and there's nothing between two and three."

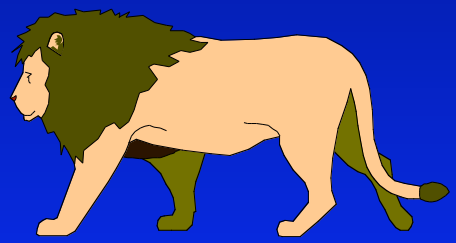
The distance between the tips of my fingers is 65 inches. Each of my arms is 24 inches long. How far is it across my chest?



# Problems

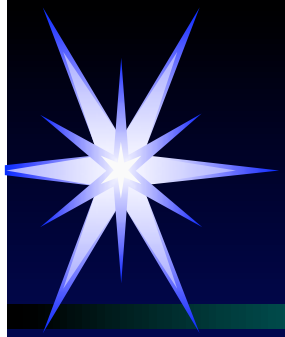
Cats

At the zoo Patty saw 21 lions and tigers. There were five more lions than tigers. How many were tigers?



A non-routine

How can you make 37 cents with seven coins?



## Mystery numbers

I am thinking of two numbers.
When I add them I get 15.
When I subtract them I get 3.
What are the two numbers?

# Learning From Mistakes

36 X 17
To make it easier, I added 4 to 36 to make 40 and 3 to 17 to make 20.

40 X 20 = 800.

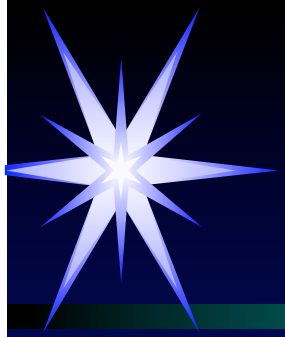
Then I subtracted 4 from 800 and then 3 from 796. My answer was 793.

# Problem Solving

- Problem solving units ‘make’ time rather ‘take’ time.
- Mathematics problems are not devoid of content
- During problem solving, students are building mathematics knowledge
- **Most mathematics can be taught through problem solving**

# Teaching Problem Solving

- Problem solving should be a significant topic in the curriculum
- Plan week-long units
- Include problem solving assessments in students' grades
- List strategies as they emerge rather than explaining them
- Use a variety of non-routine problems



# Effective Instruction

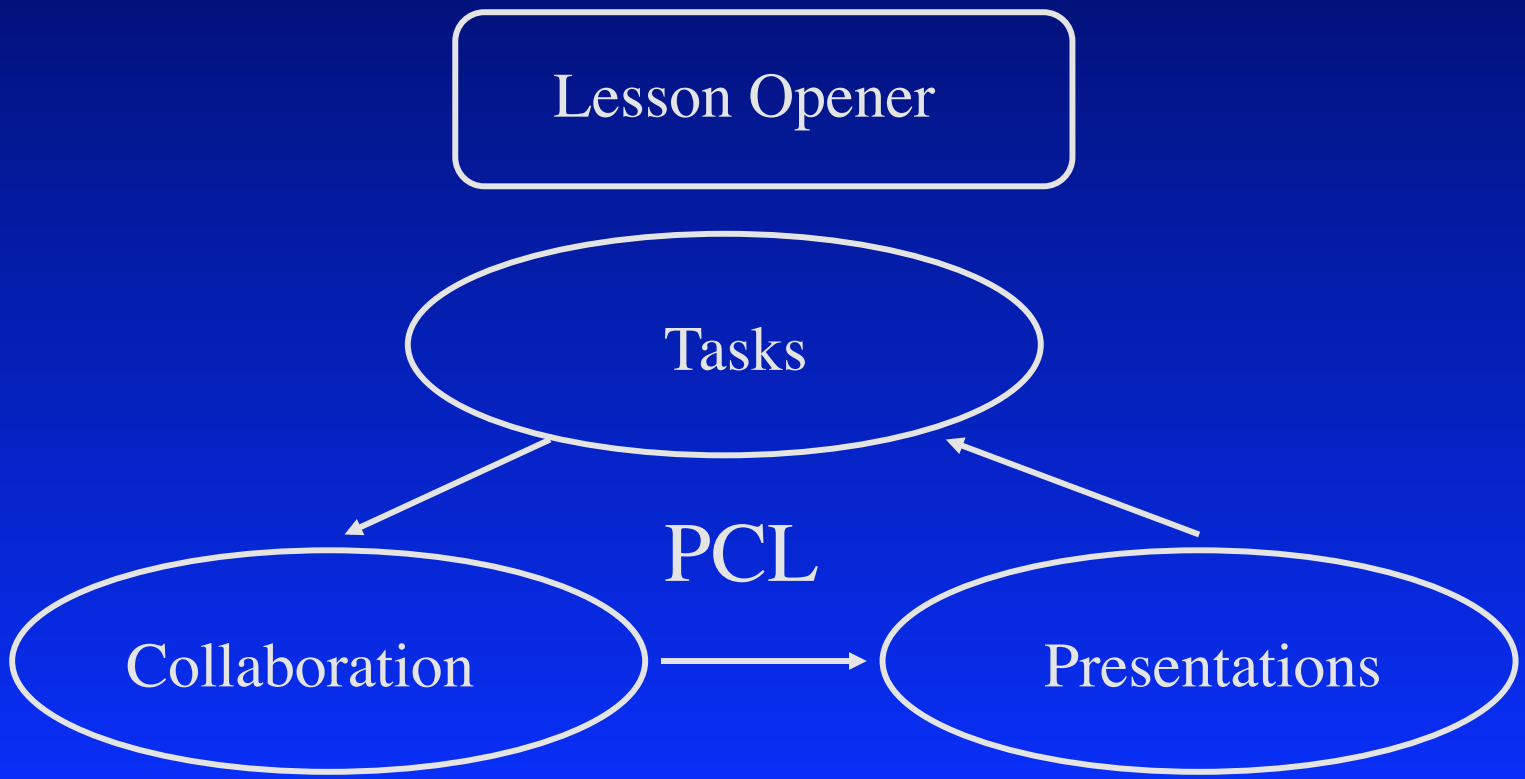
- How we teach is more important than what we teach
- The focus should be on learning
- Students should develop a sense-making orientation
- Problem solving should be a major theme

# Instructional Strategies

- There are alternatives to explain-practice
- Memorizing formulas can interfere with learning mathematics
- Students must construct meaningful ways of thinking about mathematics

# Problem Centered Instruction

- THE MODEL



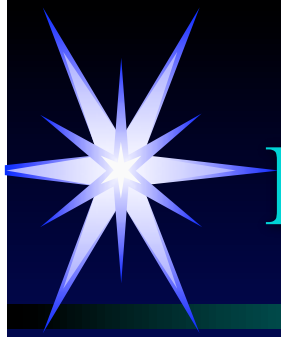
# Problem Centered Learning

- The tasks
  - should be problematic
  - require thought
  - lead somewhere mathematically

**Perturbations are seeds of learning?**

# Pairing Students

- Use like ability pairs
  - More engagement
  - More learning
  - Develops confidence
  - Better self-esteem



# Presentations

- Students present their solutions
- The Teacher helps students learn to explain
- Class determines whether it makes sense
- Teacher is nonjudgmental
- Teachers does not explain

# Problem Centered Learning

- Selecting appropriate tasks
- Negotiating social norms
- Facilitating interactions
- Being nonjudgmental
- Promoting intellectual autonomy

# Social norms

- A task requires time
- A task requires investigation
- Develop your own methods
- Work together and listen to each other
- Explain you thinking
- Expect to be puzzled

# Negotiating Social Norms

- Building the intellectual community

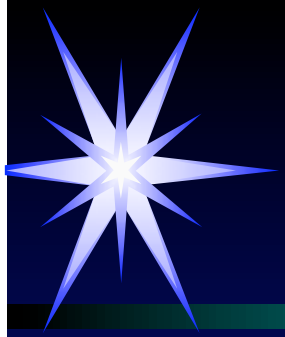
- Teacher does not explain methods to be used
- Students are responsible for determining whether answers are correct, not the teacher
- Students use solution methods that make sense to them
- Students expect to be puzzled, to be challenged

# Lesson Openers

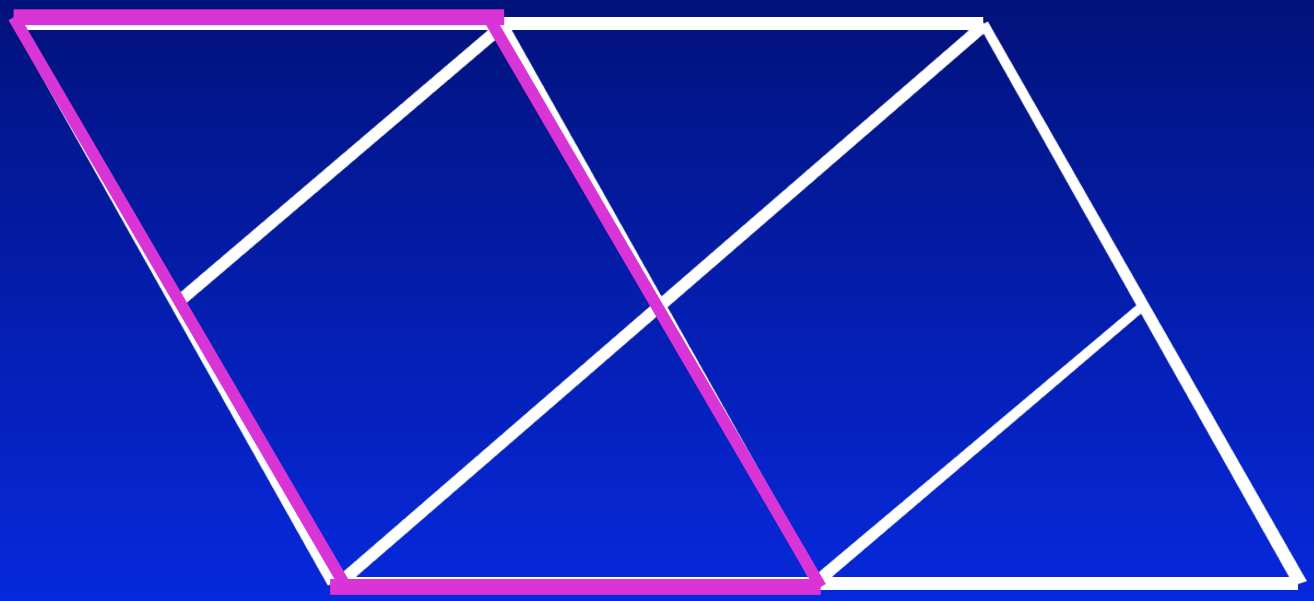
- Get the students' attention with an activity
  - Quick Draw
  - What's My Rule?
  - Mental arithmetic quickies
  - Paper Folding

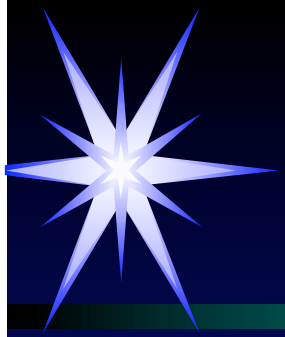

CONCEPT

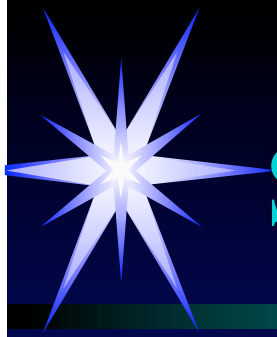
# Quick Draw



# Quick Draw







# Summary

- From dispenser of knowledge to facilitator
- Developing an atmosphere conducive to learning
- Putting students in charge of their learning

# Empty Numberline


78 ?

90 - 78 =

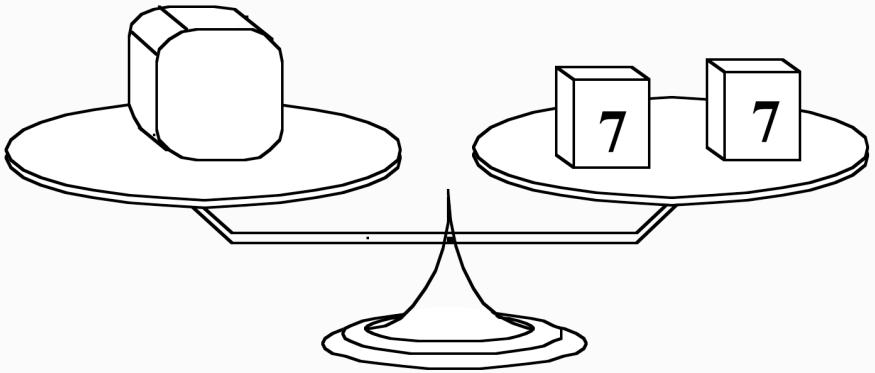
# Number Relationships


**100 ?**

**105 ?**

# Number Relationships


1250 ?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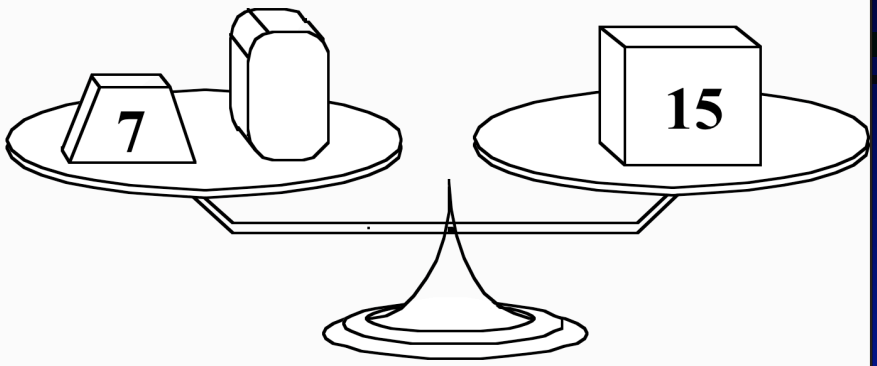
7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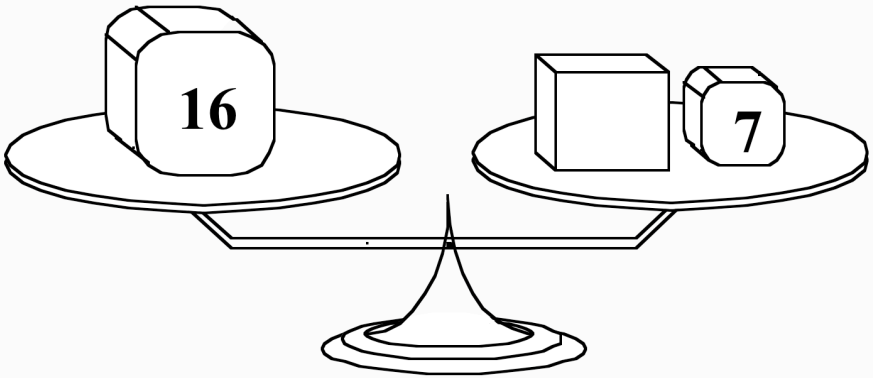
15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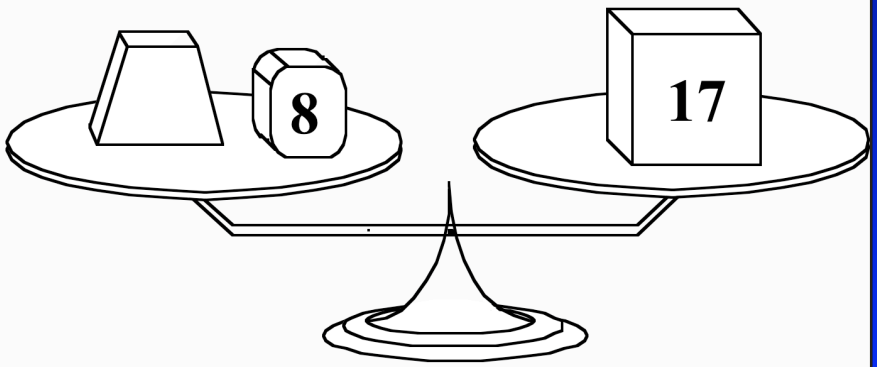
7


8
17

# Math Squares

| | |
|---|---|
| 6 | 9 |
| 11 | 4 |

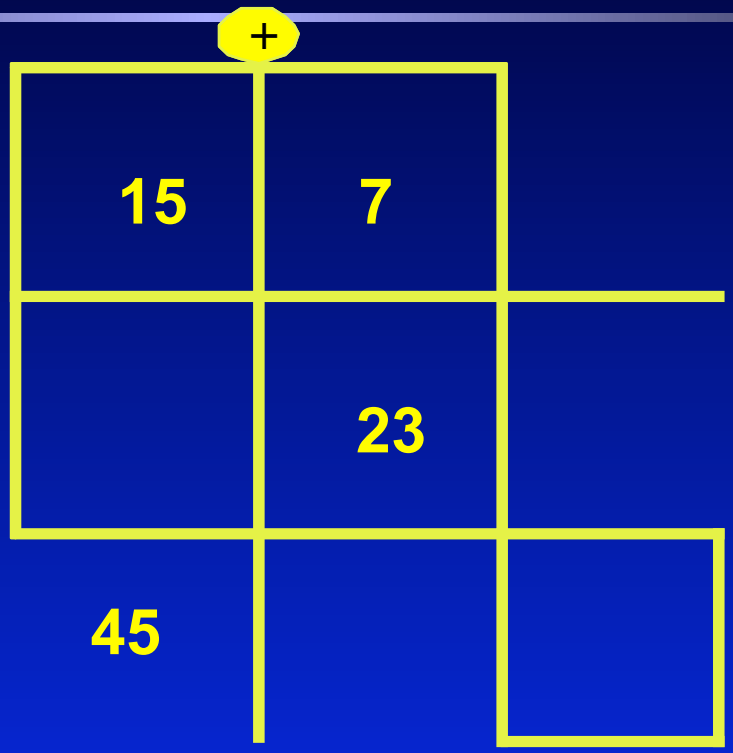
| | |
|---|---|
| | 8 |
| 12 | 15 |

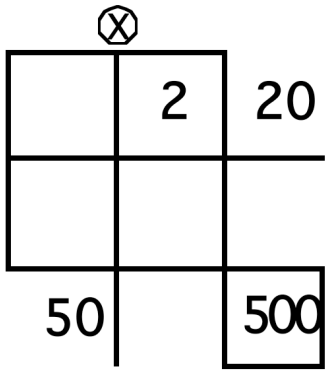
50

# Two Ways

| x | | |
|---|---|---|
| 4 | 2 | 8 |
| 3 | 5 | 15 |
| 12 | 10 | 120 |

# Two Ways

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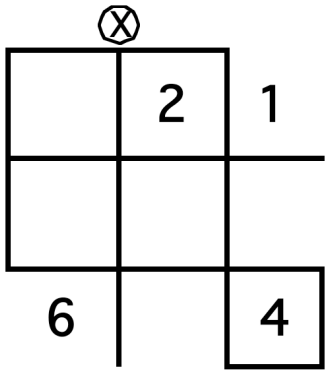
2
20
50
500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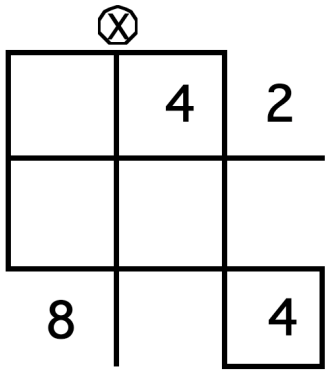
2
1
6
4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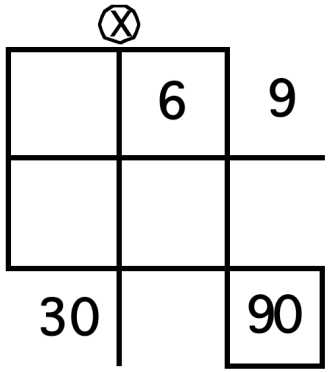
4
2
8
4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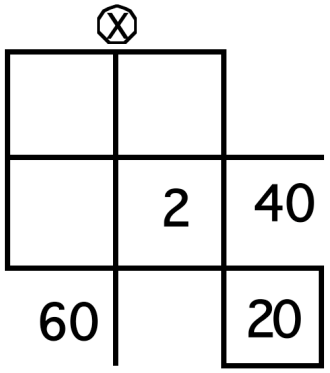
6
9
30
90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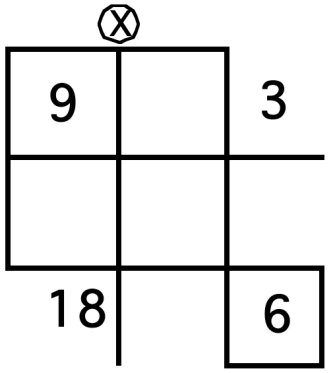
2
40
60
20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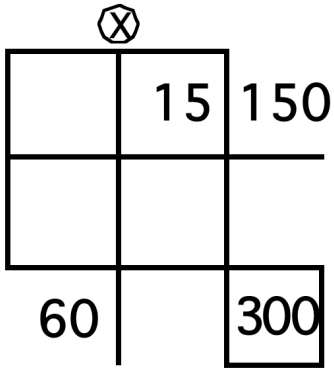
9
3
18
6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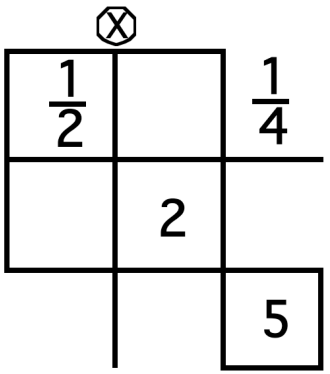
15
150
60
300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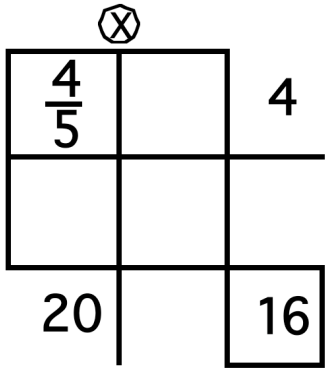
1/2
1/4
2
5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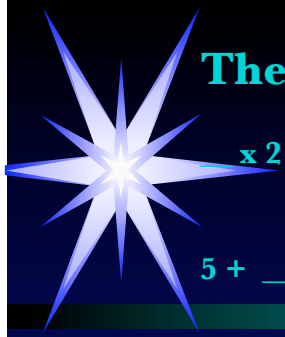

# There are 54 meaningful computations in one page of Two Ways.

__ x 2 = 20 500 ÷ 20 = __ 50 x __ = 500 50 ÷ 10 = __ 10 ÷ 2 = __

5 + __ = 25 __ x 2 = 1 1 x __ = 4 6 ÷ 1/2 = __ 6 x __ = 4

2 x __ = 3/4 12 x __ = 4 __ x 4 = 2 2 ÷ 16 = __ 8 x __ = 4

1/2 x __ = 8 30 ÷ 1 1/2 9 x __ = 90 30 x __ = 90 6 x __ = 3

20 x __ = 3 20 x __ = 10 20 ÷ 40 = __ 60 x __ = 20 1/2 ÷ 3 = __

20 ÷ 40 = 1/2 3 x __ = 1/2 40 ÷ 2 = __ 3 x __ = 60 1/6 x 2 = __

32 x __ = 4 1/4 ÷1/2 = __ 16 x __ = 1 16 ÷ 32 = __ 1/8 x 1/2 = __

__ x 15 = 150 6 x __ = 2 300 ÷ 60 = __ 10 x __ = 60 300 ÷ 150 = __

2 ÷ 6 = __ 1/2 x __ = 1/4 20 ÷ 1/4 = __ 1/2 x 2 = __ 5 x __ = 5

1/2 x __ = 5 __ x 2 + 20 1/4 __ = 5 4 ÷ 4/5 = __ 4/5 x __ = 20

16 ÷ 20 = 4 x __ = 16 25 x __ = 4 4/5 ÷ 5 = __

# Goals

- Number Sense
- Mathematical Reasoning
- Spatial Sense
- Positive Disposition